PHYS 1121 TEST 1R

(20)

1.

(a) $\underset{\sim}{r} = r\cos\theta\,\underset{\sim}{i} + r\sin\theta\,\underset{\sim}{j}$

$= r\cos(\omega t)\,\underset{\sim}{i} + r\sin(\omega t)\,\underset{\sim}{j}$ (3).

(b). $\underset{\sim}{v} = \frac{d\underset{\sim}{r}}{dt} = r(-\omega)\sin\omega t\,\underset{\sim}{i} + r\omega\cos\omega t\,\underset{\sim}{j}$

$= r\omega(-\sin\omega t\,\underset{\sim}{i} + \cos\omega t\,\underset{\sim}{j})$ (2)

$\underset{\sim}{a} = \frac{d\underset{\sim}{v}}{dt} = r\omega(-\omega\cos\omega t\,\underset{\sim}{i} - \omega\sin\omega t\,\underset{\sim}{j})$

$= -r\omega^2(\cos\omega t\,\underset{\sim}{i} + \sin\omega t\,\underset{\sim}{j})$ (2)

$= -\omega^2\underset{\sim}{r}$ (1).

(c). $\underset{\sim}{v}\cdot\underset{\sim}{r} = r^2\omega(-\cos\omega t\sin\omega t + \sin\omega t\cos\omega t) = 0$. (2).

$\underset{\sim}{v}$ is perpendicular to $\underset{\sim}{r}$. (1).

(d). $r = 1.2\,m$
$y - y_0 = 1.8\,m$
$x - x_0 = 9.0\,m$.

$y - y_0 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{2(y-y_0)/a} = \sqrt{2\times1.8/9.8} = 0.606\,s$.

$x - x_0 = v_0 t \Rightarrow v_0 = (x-x_0)/t = 9.0/0.606 = 14.9\,ms^{-1}$

$\Rightarrow$ centripetal acceleration $a_c = v^2/r = (14.9)^2/1.2 = 180\,ms^{-2}$

(6).

(e) Project onto x-axis, for instance
$\Rightarrow$ from part (b):

$a = \frac{d^2x}{dt^2} = -\omega^2 x$, $x = r\cos\omega t$

This is simple harmonic motion. (3).

2.

(a) $K = \frac{1}{2}mv^2$

Force that keeps satellite in circular orbit

$$F = -\frac{GmM_e}{r^2} = -mv^2/r$$

$$\Rightarrow K = \frac{1}{2}mv^2 = \frac{GmM_e}{2r}$$ (4).

(b) $$\frac{1}{2}mv_0^2 - \frac{GmM_e}{R_e} = \frac{1}{2}mv^2 - \frac{GmM_e}{r}$$

$$= \frac{GmM_e}{2r} - \frac{GmM_e}{r}$$

$$= -\frac{GmM_e}{2r}$$

$$\Rightarrow \frac{1}{2}\cancel{m}v_0^2 = G\cancel{m}M_e\left(\frac{1}{R_e} - \frac{1}{2r}\right)$$

$$\Rightarrow v_0 = \sqrt{2GM_e\left(\frac{1}{R_e} - \frac{1}{2r}\right)}$$ (5)

When $r \to \infty$, $v_0 \to \sqrt{2GM_e/R_e}$ escape speed from Earth (1)

(c) $E = K_A + K_B + K_e + U_{Ae} + U_{Be} + U_{AB}$, A, B – satellites

$\simeq K_A + K_B + U_{Ae} + U_{Be}$

$= 2(K + U)$, $K = K_A = K_B$; $U = U_{Ae} = U_{Be}$

$= \cancel{2}\left(-\frac{GmM_e}{\cancel{2}r}\right)$

$= -\frac{GmM_e}{r}$ (4).

(d) Cons. of momentum $mv - mv = 0 = 2mV$

$\Rightarrow$ velocity of wreckage immediately following collision $V = 0$

$\Rightarrow \quad E \simeq U_{Ae} + U_{Be} = 2U = -2\frac{Gm M_e}{r}$ (4).

(e) Wreckage falls toward Earth. As r decreases, U becomes smaller, and so the kinetic energy K gets larger; wreckage accelerates. Since the angular momentum is zero, wreckage falls directly toward centre of Earth (no rotation). (2)

3.

20

(a) $mgh = \frac{1}{2} m v_A^2$ energy cons.

$\Rightarrow v_A = \sqrt{2gh}$ 2

(b) $mgh - F_f l = \frac{1}{2} m v_B^2$

$\Rightarrow mgh - mg\mu_k l = \frac{1}{2} m v_B^2$

or can use work-energy theorem

$\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = W = -mg\mu_k l$

$\vdots$

$\Rightarrow v_B = \sqrt{2g(h - \mu_k l)}$

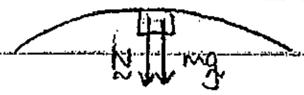
4

(c) (i) $\frac{1}{2} m v_B^2 = \frac{1}{2} m v_C^2 + mg(2r)$

$\Rightarrow v_C = \sqrt{v_B^2 - 4gr}$

$= \sqrt{2g(h - \mu_k l - 2r)}$ 3

(ii)



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(iii) Forces acting at point C: $\sum_r F = N + mg = m v_C^2 / r$

Block maintains contact with track when $N > 0$

ie $N = m v_C^2 / r - mg > 0$

$\Rightarrow m v_C^2 / r > mg$

$\Rightarrow v_C^2 > rg$

(from part (i)) $\Rightarrow 2g(h - \mu_k l - 2r) > rg$

(iii) cont.

$$\Rightarrow \quad h - \mu_k l - 2r > r/2$$

$$\Rightarrow \quad h > 5r/2 + \mu_k l .$$

$l = 3\,cm$, $r = 5\,cm$

$$\Rightarrow h > 5/2 \cdot 5 + 0.3 \cdot 3$$ 5

$$= 13\ cm$$

(d) $\frac{1}{2} m v_D^2 = \frac{1}{2} k x^2$

$v_D = v_B \Rightarrow \quad m v_B^2 = k x^2 .$

$$\Rightarrow k = m v_B^2 / x^2 = m . 2g(h - \mu_k l)/x^2$$

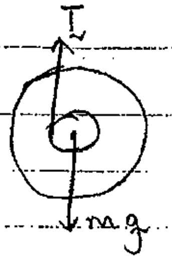
$$= 0.025 \times 2 \times 9.8 (0.18 - 0.3 \times 0.03) / (0.03)^2$$

$$= 93\ Nm^{-1}$$

3

4. (20)

(a)



The tension in the string produces a torque about the centre of mass (4)

(b) $\Sigma \tau = R_0 T = I\alpha \quad (= I a / R_0)$ (3)

(c) $\Sigma F = mg - T = ma$ (3)

(d) (i) $mg - T = ma = m\, R_0^2\, T / I$

$$\Rightarrow T(1 + mR_0^2/I) = mg,$$

$$\Rightarrow T = \frac{mg}{1 + mR_0^2/I}$$

$$= \frac{mg}{1 + \cancel{m}R_0^2/(\frac{1}{2}\cancel{m}R^2)}$$

$$= \frac{mg}{1 + 2R_0^2/R^2}$$ (3)

(ii) $a = (mg - T)/m = g - \dfrac{g}{1 + 2R_0^2/R^2}$

$$= \frac{g(\cancel{1} + 2R_0^2/R^2 - \cancel{1})}{1 + 2R_0^2/R^2}$$

$$= \frac{g\ 2R_0^2/R^2}{1 + 2R_0^2/R^2}$$ (3)

(e) Cons. energy

$\frac{1}{2} I w_0^2 = mgl \Rightarrow w_0 = \sqrt{2mgl/I}$

$\Rightarrow w_0 = \sqrt{2\cancel{m}gl/\frac{1}{2}\cancel{m}R^2} = \sqrt{4gl/R^2} = \sqrt{4 \times 9.8 \times 0.8/(0.03)^2}$

$= 187 \text{ rad s}^{-1}$ (4)

5. (20)

(a) Initial: $x_{cm} = l + L/2$

Final: $(M+50m)\,x_{cm} = (d + L/2)M + (d+L)(50m)$

Centre of mass x_{cm} is same in initial and final case (no ext. forces act).

$$\Rightarrow (M+50m)(l+L/2) = (d+L/2)M + (d+L)\,50m$$

$$= d(M+50m) + LM/2 + 50mL$$

$$\Rightarrow d = \frac{1}{M+50m}\left[(M+50m)(l+L/2) - LM/2 - 50mL\right]$$

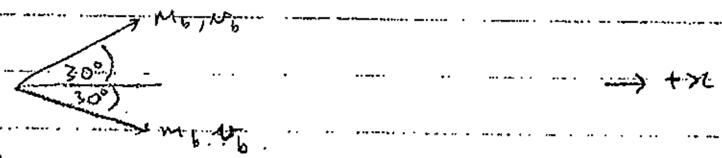
$$= \frac{1}{M+50m}\left[(M+50m)l - 25mL\right]$$

$$= \frac{1}{3500+50\times 75}\left[(3500+50\times 75)8 - 25\times 75\times 30\right]$$

$$= 24 \text{ cm}$$ (8)

(b) speed of ferry relative to the water is zero. (since centre of mass was originally at rest and there are no ext. forces) (2)

(c)



Cons. of momentum

$$0 = 2m_b v_{b_x} + (M+50m-2m_b)V$$

$$\Rightarrow V = -\frac{1}{M+50m-2m_b}\left(2m_b v_b \cos 30^\circ\right) \text{ toward croc.}$$

$$= -\frac{1}{3800+50\times 75-400}\left(2\times 200\times 20\cos 30^\circ\right)$$ (7)

$$= 0.18 \text{ ms}^{-1} \text{ away from croc.}$$

(d) Final speed according to observer

$V' = \underline{V} + v_{current}$, $v_{current} = 3 km/h$
$= 3 \times 10^3 / (60 \times 60)$
$= 0.833 ms^{-1}$

$\Rightarrow \quad V' = 0.180 + 0.833$

(3)

$= 1.0 ms^{-1}$